

For the two (2) items that follow :

Consider the function

$$f(x) = \frac{a^{[x]+x} - 1}{[x] + x}$$

where $[\cdot]$ denotes the greatest integer function.

1. What is $\lim_{x \rightarrow 0^+} f(x)$ equal to?

- (a) 1
- (b) $\ln a$
- (c) $1 - a^{-1}$
- (d) Limit does not exist

2. What is $\lim_{x \rightarrow 0^-} f(x)$ equal to?

- (a) 0
- (b) $\ln a$
- (c) $1 - a^{-1}$
- (d) Limit does not exist

For the next two (2) items that follow :

Let z_1, z_2 and z_3 be non-zero complex numbers satisfying $z^2 = i\bar{z}$, where $i = \sqrt{-1}$.

3. What is $z_1 + z_2 + z_3$ equal to?

- (a) i
 - (b) $-i$
 - (c) 0
 - (d) 1
- Handwritten note: $z_1 + 2z_2 + z_3$*

4. Consider the following statements :

- 1. $z_1 z_2 z_3$ is purely imaginary.
- 2. $z_1 z_2 + z_2 z_3 + z_3 z_1$ is purely real.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

For the next two (2) items that follow :

Given that $\log_x y, \log_z x, \log_y z$ are in GP, $xyz = 64$ and x^3, y^3, z^3 are in AP.

5. Which one of the following is correct?

x, y and z are

- (a) in AP only
- (b) in GP only
- (c) in both AP and GP
- (d) neither in AP nor in GP

6. Which one of the following is correct?

xy , yz and zx are

- (a) in AP only
- (b) in GP only
- (c) in both AP and GP
- (d) neither in AP nor in GP

For the next two (2) items that follow :

A function $f(x)$ is defined as follows :

$$f(x) = \begin{cases} x + \pi & \text{for } x \in [-\pi, 0) \\ \pi \cos x & \text{for } x \in \left[0, \frac{\pi}{2}\right] \\ \left(x - \frac{\pi}{2}\right)^2 & \text{for } x \in \left[\frac{\pi}{2}, \pi\right] \end{cases}$$

9. Consider the following statements :

1. The function $f(x)$ is continuous at $x = 0$.
2. The function $f(x)$ is continuous at $x = \frac{\pi}{2}$.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

For the next two (2) items that follow :

Let z be a complex number satisfying

$$\left| \frac{z-4}{z-8} \right| = 1 \text{ and } \left| \frac{z}{z-2} \right| = \frac{3}{2}$$

7. What is $|z|$ equal to?

- (a) 6
- (b) 12
- (c) 18
- (d) 36

$f(x) = \pi$
 $\lim_{x \rightarrow 0^+} f(x) = \pi$
 $\lim_{x \rightarrow 0^-} f(x) = \pi$

$\lim_{x \rightarrow \frac{\pi}{2}} f(x) = 0$

$\lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = \left(\frac{\pi}{2} - \frac{\pi}{2}\right)^2 = 0$
 $\lim_{x \rightarrow \frac{\pi}{2}^+} f(x) = \left(\frac{\pi}{2} - \frac{\pi}{2}\right)^2 = 0$

8. What is $\left| \frac{z-6}{z+6} \right|$ equal to?

- (a) 3
- (b) 2
- (c) 1
- (d) 0

10. Consider the following statements :

1. The function $f(x)$ is differentiable at $x = 0$.
2. The function $f(x)$ is differentiable at $x = \frac{\pi}{2}$.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

For the next two (2) items that follow :

For the next three (3) items that follow :

Let α and β ($\alpha < \beta$) be the roots of the equation $x^2 + bx + c = 0$, where $b > 0$ and $c < 0$.

11. Consider the following :

1. $\beta < -\alpha$
2. $\beta < |\alpha|$

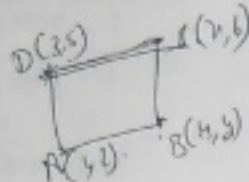
Which of the above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2



Handwritten calculations for question 12:

$$AB = 3 = \sqrt{(4-4)^2 + (3-1)^2}$$

$$BC = \sqrt{(x-4)^2 + (6-3)^2} = \sqrt{(x-4)^2 + 9}$$

$$AC = \sqrt{(x-3)^2 + (6-5)^2} = \sqrt{(x-3)^2 + 1}$$

$$BD = \sqrt{(3-4)^2 + (5-1)^2} = \sqrt{1 + 16} = \sqrt{17}$$

12. Consider the following :

1. $\alpha + \beta + \alpha\beta > 0$
2. $\alpha^2\beta + \beta^2\alpha > 0$

Which of the above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Handwritten calculation for question 12:

$$b + c > 0$$

Handwritten calculation for question 12:

$$b > 0$$

Handwritten calculation for question 12:

$$c < 0$$

Handwritten calculation for question 12:

$$\alpha^2\beta + \beta^2\alpha = 0$$

Handwritten calculation for question 12:

$$\alpha\beta(\alpha + \beta) = -c$$

Handwritten calculation for question 12:

$$c(b) > 0$$

Consider a parallelogram whose vertices are $A(1, 2)$, $B(4, y)$, $C(x, 6)$ and $D(3, 5)$ taken in order.

13. What is the value of $AC^2 - BD^2$?

(a) 25

(b) 30

(c) 36

(d) 40

14. What is the point of intersection of the diagonals?

(a) $(\frac{7}{2}, 4)$

(b) (3, 4)

(c) $(\frac{7}{2}, 5)$

(d) (3, 5)

15. What is the area of the parallelogram?

(a) $\frac{7}{2}$ square units

(b) 4 square units

(c) $\frac{11}{2}$ square units

(d) 7 square units

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16. Let $f(x)$ and $g(x)$ be twice differentiable functions on $[0, 2]$ satisfying $f''(x) = g''(x)$, $f'(1) = 4$, $g'(1) = 6$, $f(2) = 3$ and $g(2) = 9$. Then what is $f(x) - g(x)$ at $x = 4$ equal to?

$(x) = g''(x)$

- (a) -10
- (b) -6
- (c) -4
- (d) 2

$f'(1) = 4 = f'$
 $f(2) = 3 = \frac{3}{2}(2)^2$
 $f''(x) = g''(x)$

For the next two (2) items that follow :

Consider the function

$$f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

where p is a constant.

19. What is the value of $f'(0)$?

- (a) p^3
- (b) $3p^3$
- (c) $6p^3$
- (d) $-6p^3$

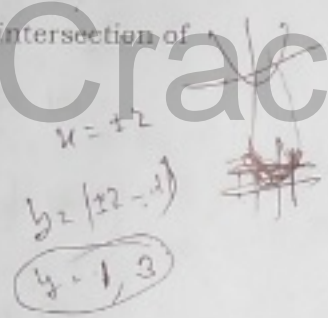
For the next two (2) items that follow :

Consider the curves

$$y = |x - 1| \text{ and } |x| = 2$$

17. What is/are the point(s) of intersection of the curves?

- (a) $(-2, 3)$ only
- (b) $(2, 1)$ only
- (c) $(-2, 3)$ and $(2, 1)$
- (d) Neither $(-2, 3)$ nor $(2, 1)$



20. What is the value of p for which $f''(0) = 0$?

- (a) $-\frac{1}{6}$ or 0
- (b) -1 or 0
- (c) $-\frac{1}{6}$ or 1
- (d) -1 or 1

For the next two (2) items that follow :

Consider a triangle ABC in which

$$\cos A + \cos B + \cos C = \sqrt{3} \sin \frac{\pi}{3}$$

18. What is the area of the region bounded by the curves and x -axis?

- (a) 3 square units
- (b) 4 square units
- (c) 5 square units
- (d) 6 square units



21. What is the value of $\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$?

- (a) $\frac{1}{2}$
- (b) $\frac{1}{4}$
- (c) $\frac{1}{8}$
- (d) $\frac{1}{16}$

$\int (x-1) dx$

22. What is the value of

$$\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{C+A}{2}\right)?$$

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) $\frac{1}{16}$

(d) None of the above

Handwritten notes: $\tan \alpha + \tan \beta = -b/c$

$(1, 3)$

$(-4, -1)$

$\sqrt{25+16}$
 $= 7$

For the next two (2) items that follow :

Consider the two circles

$$(x-1)^2 + (y-3)^2 = r^2 \text{ and } x^2 + y^2 - 8x + 2y + 8 = 0$$

25. What is the distance between the centres of the two circles?

(a) 5 units

(b) 6 units

(c) 8 units

(d) 10 units

$(1, 3)$

$(4, -1)$

$\sqrt{9+16} = 5$

For the next two (2) items that follow :

Given that $\tan \alpha$ and $\tan \beta$ are the roots of the equation $x^2 + bx + c = 0$ with $b \neq 0$.

23. What is $\tan(\alpha + \beta)$ equal to?

(a) $b(c-1)$

(b) $c(b-1)$

(c) $c(b-1)^{-1}$

(d) $b(c-1)^{-1}$

Handwritten notes:
 $\tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \frac{b(c-1)^{-1}}{b(c-1)^{-1}} = 1$

26. If the circles intersect at two distinct points, then which one of the following is correct?

(a) $r = 1$

(b) $1 < r < 2$

(c) $r = 2$

(d) $2 < r < 8$

For the next two (2) items that follow :

Consider the two lines

$$x + y + 1 = 0 \text{ and } 3x + 2y + 1 = 0$$

24. What is $\sin(\alpha + \beta) \sec \alpha \sec \beta$ equal to?

(a) b

(b) $-b$

(c) c

(d) $-c$

Handwritten notes:
 $n = -(y+1)$
 $-3(y+1) + 2y + 1 = 0$
 $-3y - 3 + 2y + 1 = 0$
 $-y - 2 = 0$

27. What is the equation of the line passing through the point of intersection of the given lines and parallel to x -axis?

(a) $y + 1 = 0$

(b) $y - 1 = 0$

(c) $y - 2 = 0$

(d) $y + 2 = 0$

28. What is the equation of the line passing through the point of intersection of the given lines and parallel to y -axis?

(a) $x+1=0$

(b) $x-1=0$

(c) $x-2=0$

(d) $x+2=0$

For the next two (2) items that follow :

Consider the equation

$$k \sin x + \cos 2x = 2k - 7$$

29. If the equation possesses solution, then what is the minimum value of k ?

(a) 1

(b) 2

(c) 4

(d) 6

30. If the equation possesses solution, then what is the maximum value of k ?

(a) 1

(b) 2

(c) 4

(d) 6

For the next two (2) items that follow :

Consider the functions

$$f(x) = xg(x) \text{ and } g(x) = \left[\frac{1}{x} \right]$$

where $[\cdot]$ is the greatest integer function.

31. What is $\int_{\frac{1}{3}}^1 g(x) dx$ equal to?

(a) $\frac{1}{6}$

(b) $\frac{1}{3}$

(c) $\frac{5}{18}$

(d) $\frac{5}{36}$

32. What is $\int_{\frac{1}{3}}^1 f(x) dx$ equal to?

(a) $\frac{37}{72}$

(b) $\frac{2}{3}$

(c) $\frac{17}{72}$

(d) $\frac{37}{144}$

For the next five (5) items that follow :

Consider the function

$$f(x) = |x-1| + x^2$$

where $x \in \mathbb{R}$.

33. Which one of the following statements is correct?

(a) $f(x)$ is continuous but not differentiable at $x=0$

(b) $f(x)$ is continuous but not differentiable at $x=1$

(c) $f(x)$ is differentiable at $x=1$

(d) $f(x)$ is not differentiable at $x=0$ and $x=1$

Handwritten notes:
 $k \cos x - 2 \sin 2x = 2k - 7$
 $-2 \sin 2x = (2 - k \cos x) k$
 $k \cos x - 2 \sin 2x = 0$
 $k = \frac{2 \sin 2x}{\cos x} = \frac{4 \sin x \cos x}{\cos x} = 4 \sin x$

Handwritten notes for Q30:
 $f(x) = |x-1| + x^2$
 $f(0) = 1$
 $\lim_{h \rightarrow 0^+} = f(h-1) + h^2 = 1$

34. Which one of the following statements is correct?

(a) $f(x)$ is increasing in $(-\infty, \frac{1}{2})$ and decreasing in $(\frac{1}{2}, \infty)$

(b) $f(x)$ is decreasing in $(-\infty, \frac{1}{2})$ and increasing in $(\frac{1}{2}, \infty)$

(c) $f(x)$ is increasing in $(-\infty, 1)$ and decreasing in $(1, \infty)$

(d) $f(x)$ is decreasing in $(-\infty, 1)$ and increasing in $(1, \infty)$

35. Which one of the following statements is correct?

(a) $f(x)$ has local minima at more than one point in $(-\infty, \infty)$

(b) $f(x)$ has local maxima at more than one point in $(-\infty, \infty)$

(c) $f(x)$ has local minimum at one point only in $(-\infty, \infty)$

(d) $f(x)$ has neither maxima nor minima in $(-\infty, \infty)$

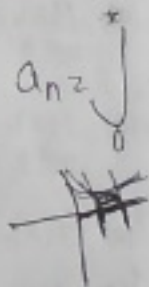
36. What is the area of the region bounded by x -axis, the curve $y = f(x)$ and the two ordinates $x = \frac{1}{2}$ and $x = 1$?

(a) $\frac{5}{12}$ square unit

(b) $\frac{5}{6}$ square unit

(c) $\frac{7}{6}$ square units

(d) 2 square units



37. What is the area of the region bounded by x -axis, the curve $y = f(x)$ and the two ordinates $x = 1$ and $x = \frac{3}{2}$?

(a) $\frac{5}{12}$ square unit

(b) $\frac{7}{12}$ square unit

(c) $\frac{2}{3}$ square unit

(d) $\frac{11}{12}$ square unit

For the next two (2) items that follow :

Given that

$$a_n = \int_0^{\pi} \frac{\sin^2 \{(n+1)x\}}{\sin 2x} dx$$

38. Consider the following statements :

1. The sequence $\{a_{2n}\}$ is in AP with common difference zero.

2. The sequence $\{a_{2n+1}\}$ is in AP with common difference zero.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

39. What is $a_{n-1} - a_{n-4}$ equal to?

(a) -1

(b) 0

(c) 1

(d) 2

For the next two (2) items that follow :

Consider the equation $x + |y| = 2y$.

40. Which of the following statements are not correct?

1. y as a function of x is not defined for all real x .
2. y as a function of x is not continuous at $x = 0$.
3. y as a function of x is differentiable for all x .

Select the correct answer using the code given below.

- (a) 1 and 2 only
 (b) 2 and 3 only
 (c) 1 and 3 only
 (d) 1, 2 and 3

41. What is the derivative of y as a function of x with respect to x for $x < 0$?

- (a) 2 (b) 1
 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

$x + |y| = 2y$
 $x = 2y - |y|$
 $x = 2y - (-y)$
 $x = 3y$
 $y = \frac{x}{3}$
 $y' = \frac{1}{3}$

For the next two (2) items that follow :

Consider the lines

$$y = 3x, y = 6x \text{ and } y = 9$$



42. What is the area of the triangle formed by these lines?

- (a) $\frac{27}{4}$ square units
 (b) $\frac{27}{2}$ square units
 (c) $\frac{19}{4}$ square units
 (d) $\frac{19}{2}$ square units

$(1, 3)$
 $(1, 6)$
 $(0, 9)$
 $\frac{2}{3} \times 6$

43. The centroid of the triangle is at which one of the following points?

- (a) (3, 6)
 (b) $(\frac{3}{2}, 6)$
 (c) (3, 3)
 (d) $(\frac{3}{2}, 9)$

For the next two (2) items that follow :

Consider the function

$$f(x) = (x - 1)^2 (x + 1)(x - 2)^3$$

44. What is the number of points of local minima of the function $f(x)$?

- (a) None
 (b) One
 (c) Two
 (d) Three

45. What is the number of points of local maxima of the function $f(x)$?

- (a) None
 (b) One
 (c) Two
 (d) Three

46. Suppose ω is a cube root of unity with $\omega \neq 1$. Suppose P and Q are the points on the complex plane defined by ω and ω^2 . If O is the origin, then what is the angle between OP and OQ ?

- (a) 60°
- (b) 90°
- (c) 120°
- (d) 150°

47. Suppose there is a relation $*$ between the positive numbers x and y given by $x * y$ if and only if $x \leq y^2$. Then which one of the following is correct?

- (a) $*$ is reflexive but not transitive and symmetric
- (b) $*$ is transitive but not reflexive and symmetric
- (c) $*$ is symmetric and reflexive but not transitive
- (d) $*$ is symmetric but not reflexive and transitive

48. If $x^2 - px + 4 > 0$ for all real values of x , then which one of the following is correct?

- (a) $|p| < 4$
- (b) $|p| \leq 4$
- (c) $|p| > 4$
- (d) $|p| \geq 4$

$$(x^2 - px + 4) > 0$$

49. If $z = x + iy = \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^{-25}$, where $i = \sqrt{-1}$, then what is the fundamental amplitude of $\frac{z - \sqrt{2}}{z - i\sqrt{2}}$?

- (a) π
- (b) $\frac{\pi}{2}$
- (c) $\frac{\pi}{3}$
- (d) $\frac{\pi}{4}$

50. If

$$f(x_1) - f(x_2) = f\left(\frac{x_1 - x_2}{1 - x_1 x_2}\right)$$

for $x_1, x_2 \in (-1, 1)$, then what is $f(x)$ equal to?

- (a) $\ln\left(\frac{1-x}{1+x}\right)$
- (b) $\ln\left(\frac{2+x}{1-x}\right)$
- (c) $\tan^{-1}\left(\frac{1-x}{1+x}\right)$
- (d) $\tan^{-1}\left(\frac{1+x}{1-x}\right)$

51. What is the range of the function

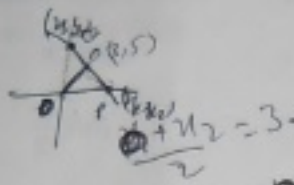
$$y = \frac{x^2}{1+x^2}$$

where $x \in \mathbb{R}$?

- (a) $[0, 1)$
- (b) $[0, 1]$
- (c) $(0, 1)$
- (d) $(0, 1]$

52. A straight line intersects x and y axes at P and Q respectively. If $(3, 5)$ is the middle point of PQ , then what is the area of the triangle OPQ ?

- (a) 12 square units
- (b) 15 square units
- (c) 20 square units
- (d) 30 square units



53. If a circle of radius b units with centre at $(0, b)$ touches the line $y = x - \sqrt{2}$, then what is the value of b ?

- (a) $2 + \sqrt{2}$
- (b) $2 - \sqrt{2}$
- (c) $2\sqrt{2}$
- (d) $\sqrt{2}$



Handwritten notes for question 53: $y_1 = 10$, $x_2 = 6$, $(y-10) = \frac{6}{-10}(x-6)$, $10y - 100 = 6x - 36$

56. Consider the following statements :

1. $f(\theta) = 2$ has no solution.
2. $f(\theta) = \frac{7}{2}$ has a solution.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

For the next three (3) items that follow :

Consider the function

$f(\theta) = 4(\sin^2 \theta + \cos^4 \theta)$

54. What is the maximum value of the function $f(\theta)$?

- (a) 1
- (b) 2
- (c) 3
- (d) 4

Handwritten notes for question 54: $f'(\theta) = 4(2\sin\theta\cos\theta + 4\cos^3\theta(-\sin\theta))$, $4\sin\theta(2 + 4\cos^2\theta)$

55. What is the minimum value of the function $f(\theta)$?

- (a) 0
- (b) 1
- (c) 2
- (d) 3

Handwritten notes for question 55: $8\sin\theta\cos\theta(1 + 2\cos^2\theta)$, $8\sin\theta\cos\theta = 0$, $\theta = \frac{\pi}{4}$, $1 + 2\cos^2\theta = 0$, $\theta = 0$

Consider the curves

$f(x) = x|x| - 1$ and $g(x) = \begin{cases} \frac{3x}{2}, & x > 0 \\ 2x, & x \leq 0 \end{cases}$

For the next two (2) items that follow :

57. Where do the curves intersect?

- (a) At $(2, 3)$ only
- (b) At $(-1, -2)$ only
- (c) At $(2, 3)$ and $(-1, -2)$
- (d) Neither at $(2, 3)$ nor at $(-1, -2)$

58. What is the area bounded by the curves?

- (a) $\frac{17}{6}$ square units
- (b) $\frac{8}{3}$ square units
- (c) 2 square units
- (d) $\frac{1}{3}$ square unit

For the next two (2) items that follow :

Consider the function

$$f(x) = \frac{27(x^{2/3} - x)}{4}$$

59. How many solutions does the function $f(x) = 1$ have?

- (a) One
- (b) Two
- (c) Three
- (d) Four

60. How many solutions does the function $f(x) = -1$ have?

- (a) One
- (b) Two
- (c) Three
- (d) Four

61. A fair coin is tossed 100 times. What is the probability of getting tails an odd number of times?

- ~~(a)~~ $\frac{1}{2}$
- (b) $\frac{3}{8}$
- (c) $\frac{1}{4}$
- (d) $\frac{1}{8}$

62. What is the number of ways in which 3 holiday travel tickets are to be given to 10 employees of an organization, if each employee is eligible for any one or more of the tickets?

- ~~(a)~~ 60
 - (b) 120
 - (c) 500
 - (d) 1000
- Handwritten notes: $3 \times 10 + 2 \times 10^2$*

63. If one root of the equation

$$(l-m)x^2 + lx + 1 = 0$$

is double the other and l is real, then what is the greatest value of m ?

- (a) $-\frac{9}{8}$
- (b) $\frac{9}{8}$
- (c) $-\frac{8}{9}$
- (d) $\frac{8}{9}$

64. What is the number of four-digit decimal numbers (< 1) in which no digit is repeated?

- (a) 3024
 - (b) 4536
 - (c) 5040
 - (d) None of the above
- Handwritten note: $0.9! \times 8! \times 2 \times 4!$*

65. What is a vector of unit length orthogonal to both the vectors $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} + 3\hat{j} - \hat{k}$?

(a) $\frac{-4\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{26}}$

(b) $\frac{-4\hat{i} + 3\hat{j} + \hat{k}}{\sqrt{26}}$

(c) $\frac{-3\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{14}}$

(d) $\frac{-3\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{14}}$

Handwritten solution for Q65:

$$A^3 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

68. Consider the following in respect of the matrix $A = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$:

1. $A^2 = -A$

2. $A^3 = 4A$

Which of the above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

66. If \vec{a} , \vec{b} and \vec{c} are the position vectors of the vertices of an equilateral triangle whose orthocentre is at the origin, then which one of the following is correct?

(a) $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

(b) $\vec{a} + \vec{b} + \vec{c} = \text{unit vector}$

(c) $\vec{a} + \vec{b} = \vec{c}$

(d) $\vec{a} = \vec{b} + \vec{c}$

Handwritten solution for Q66:

$$\begin{vmatrix} i & j & k \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$

$$= i(4-6) - j(12+2) + k(-9-1)$$

$$= -2i - 14j - 10k$$

69. Which of the following determinants have value zero?

1. $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$

2. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

3. $\begin{vmatrix} 0 & a & b \\ -c & 0 & a \\ -b & -a & 0 \end{vmatrix} = -c(ab) + b(ac) - abc = abc - abc = 0$

67. What is the area of the parallelogram having diagonals $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$?

(a) $5\sqrt{5}$ square units

(b) $4\sqrt{5}$ square units

(c) $5\sqrt{3}$ square units

(d) $15\sqrt{2}$ square units

Handwritten solution for Q67:

$$\frac{1}{2} \sqrt{264}$$

$$= \frac{1}{2} \sqrt{4 \cdot 66}$$

$$= \frac{1}{2} \cdot 2 \sqrt{66} = \sqrt{66}$$

Handwritten solution for Q67:

$$\frac{1}{2} \sqrt{232}$$

$$= \frac{1}{2} \sqrt{8 \cdot 29}$$

$$= \frac{1}{2} \cdot 2\sqrt{29} = \sqrt{29}$$

Select the correct answer using the code given below.

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

70. What is the acute angle between the lines represented by the equations $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$?

(a) 30°

(b) 45°

(c) 60°

(d) 75°

Ans = $\sqrt{3} + \sqrt{3}$
 $= \sqrt{6}$
 $-\sqrt{3}i + j - 5 \cdot k$
 $-j + \sqrt{3}y + 6x$
 $\frac{\sqrt{3} + \sqrt{3} - 30}{\sqrt{29} \sqrt{40}} = \frac{2\sqrt{3} - 40}{2\sqrt{110}}$

71. The system of linear equations $kx + y + z = 1$, $x + ky + z = 1$ and $x + y + kz = 1$ has a unique solution under which one of the following conditions?

(a) $k \neq 1$ and $k \neq -2$

(b) $k \neq 1$ and $k \neq 2$

(c) $k \neq -1$ and $k \neq -2$

(d) $k \neq -1$ and $k \neq 2$

Ans = $\sqrt{3} + \sqrt{3} - 30$

72. What is the number of different messages that can be represented by three 0's and two 1's?

(a) 10

(b) 9

(c) 8

(d) 7

000 11

110 00

101 00

100 10

1000 1

0100 1

0010 1

0001 1

73. If $\log_a(ab) = x$, then what is $\log_b(ab)$ equal to?

(a) $\frac{1}{x}$

(b) $\frac{x}{x+1}$

(c) $\frac{x}{1-x}$

(d) $\frac{x}{x-1}$

Ans = $\log_a(ab) = x$
 $\log_a a + \log_a b = x$
 $1 + \log_a b = x$
 $\log_a b = x - 1$
 $\log_b(ab) = \frac{1 + \log_a b}{\log_a b} = \frac{1 + x - 1}{x - 1} = \frac{x}{x - 1}$

74. If

$$y = \log_{10} x + \log_x 10 + \log_x x + \log_{10} 10$$

then what is

$$\left(\frac{dy}{dx}\right)_{x=10}$$

equal to?

(a) 10

(b) 2

(c) 1

(d) 0

Ans = $\frac{1}{x} + \frac{1}{10} + 1 + 1 = \frac{1}{x} + \frac{1}{10} + 2$

75. Suppose ω_1 and ω_2 are two distinct cube roots of unity different from 1. Then what is $(\omega_1 - \omega_2)^2$ equal to?

(a) 3

(b) 1

(c) -1

(d) -3

76. What is the mean deviation from the mean of the numbers 10, 9, 21, 16, 24 ?

- (a) 5.2
- (b) 5.0
- (c) 4.5
- (d) 4.0

1, 2, 3
2, 1, 2
2, 2, 1

77. Three dice are thrown simultaneously. What is the probability that the sum on the three faces is at least 5?

- (a) $\frac{17}{18}$
- (b) $\frac{53}{54}$
- (c) $\frac{103}{108}$
- (d) $\frac{215}{216}$

$1 - \frac{1}{216}$
1 - 1/216
215/216
1 - 1/216
215/216
1 - 1/216
215/216
1 - 1/216
215/216

78. Two independent events A and B have $P(A) = \frac{1}{3}$ and $P(B) = \frac{3}{4}$. What is the probability that exactly one of the two events A or B occurs?

- (a) $\frac{1}{4}$
- (b) $\frac{5}{6}$
- (c) $\frac{5}{12}$
- (d) $\frac{7}{12}$

79. A coin is tossed three times. What is the probability of getting head and tail alternately?

- (a) $\frac{1}{8}$
- (b) $\frac{1}{4}$
- (c) $\frac{1}{2}$
- (d) $\frac{3}{4}$

H T H
T H T

80. If the total number of observations is 20, $\sum x_i = 1000$ and $\sum x_i^2 = 84000$, then what is the variance of the distribution?

- (a) 1500
- (b) 1600
- (c) 1700
- (d) 1800

81. A card is drawn from a well-shuffled deck of 52 cards. What is the probability that it is queen of spade?

- (a) $\frac{1}{52}$
- (b) $\frac{1}{13}$
- (c) $\frac{1}{4}$
- (d) $\frac{1}{8}$

82. If two dice are thrown, then what is the probability that the sum on the two faces is greater than or equal to 4?

(a) $\frac{13}{18}$

(b) $\frac{5}{6}$

(c) $\frac{11}{12}$

(d) $\frac{35}{36}$

Handwritten notes for Q82:
 $1, 1, \frac{1}{36}$
 $1, 2, \frac{1}{36}$
 $2, 1, \frac{1}{36}$
 $1, 3, \frac{1}{36}$
 $3, 1, \frac{1}{36}$
 $1, 4, \frac{1}{36}$
 $4, 1, \frac{1}{36}$
 $1, 5, \frac{1}{36}$
 $5, 1, \frac{1}{36}$
 $1, 6, \frac{1}{36}$
 $6, 1, \frac{1}{36}$
 $2, 2, \frac{2}{36}$
 $3, 2, \frac{2}{36}$
 $2, 3, \frac{2}{36}$
 $4, 2, \frac{2}{36}$
 $2, 4, \frac{2}{36}$
 $5, 2, \frac{2}{36}$
 $2, 5, \frac{2}{36}$
 $6, 2, \frac{2}{36}$
 $3, 3, \frac{3}{36}$
 $4, 3, \frac{3}{36}$
 $3, 4, \frac{3}{36}$
 $5, 3, \frac{3}{36}$
 $3, 5, \frac{3}{36}$
 $6, 3, \frac{3}{36}$
 $4, 4, \frac{4}{36}$
 $5, 4, \frac{4}{36}$
 $4, 5, \frac{4}{36}$
 $6, 4, \frac{4}{36}$
 $5, 5, \frac{5}{36}$
 $6, 5, \frac{5}{36}$
 $5, 6, \frac{5}{36}$
 $6, 6, \frac{6}{36}$

83. A certain type of missile hits the target with probability $p = 0.3$. What is the least number of missiles should be fired so that there is at least an 80% probability that the target is hit?

(a) 5

(b) 6

(c) 7

(d) None of the above

84. For two mutually exclusive events A and B, $P(A) = 0.2$ and $P(\bar{A} \cap B) = 0.3$. What is $P(A | (A \cup B))$ equal to?

(a) $\frac{1}{2}$

(b) $\frac{2}{5}$

(c) $\frac{2}{7}$

(d) $\frac{2}{3}$



85. What is the probability of 5 Sundays in the month of December?

(a) $\frac{1}{7}$

(b) $\frac{2}{7}$

(c) $\frac{3}{7}$

(d) None of the above

86. If m is the geometric mean of

$\left(\frac{y}{z}\right)^{\log(yz)}$, $\left(\frac{z}{x}\right)^{\log(zx)}$ and $\left(\frac{x}{y}\right)^{\log(xy)}$

then what is the value of m ?

(a) 1

(b) 3

(c) 6

(d) 9

87. A point is chosen at random inside a rectangle measuring 6 inches by 5 inches. What is the probability that the randomly selected point is at least one inch from the edge of the rectangle?

(a) $\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\frac{1}{4}$

(d) $\frac{2}{5}$

88. The mean of the series x_1, x_2, \dots, x_n is \bar{X} . If x_2 is replaced by λ , then what is the new mean?

(a) $\bar{X} - x_2 + \lambda$

(b) $\frac{\bar{X} - x_2 - \lambda}{n}$

(c) $\frac{\bar{X} - x_2 + \lambda}{n}$

(d) $\frac{n\bar{X} - x_2 + \lambda}{n}$

89. For the data

3, 5, 1, 6, 5, 9, 5, 2, 8, 6

the mean, median and mode are x , y and z respectively. Which one of the following is correct?

(a) $x = y \neq z$

(b) $x \neq y = z$

(c) $x \neq y \neq z$

(d) $x = y = z$

90. Consider the following statements in respect of a histogram :

1. The total area of the rectangles in a histogram is equal to the total area bounded by the corresponding frequency polygon and the x -axis.

2. When class intervals are unequal in a frequency distribution, the area of the rectangle is proportional to the frequency.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

91. Consider the following -

1. There exists $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which $\tan^{-1}(\tan \theta) \neq \theta$.

2. $\sin^{-1}\left(\frac{1}{3}\right) - \sin^{-1}\left(\frac{1}{5}\right)$
 $= \sin^{-1}\left(\frac{2\sqrt{2}(\sqrt{3}-1)}{15}\right)$

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

92. Consider the following statements :

1. $\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = \pi$

2. There exist $x, y \in [-1, 1]$, where $x \neq y$ such that $\sin^{-1} x + \cos^{-1} y = \frac{\pi}{2}$.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

93. What are the order and degree respectively of the differential equation whose solution is $y = cx + c^2 - 3c^{3/2} + 2$, where c is a parameter?

- (a) 1, 2
- (b) 2, 2
- (c) 1, 3
- (d) 1, 4

94. What is

$$\int_{-2}^2 x dx - \int_{-2}^2 [x] dx$$

equal to, where $[\cdot]$ is the greatest integer function?

- (a) 0
- (b) 1
- (c) 2
- (d) 4

0.3125 x 2

95. If

$$\int_{-2}^5 f(x) dx = 4 \text{ and } \int_0^5 [1 + f(x)] dx = 7$$

then what is $\int_{-2}^0 f(x) dx$ equal to?

- (a) -3
- (b) 2
- (c) 3
- (d) 5

$$\int_{-2}^0 f(x) dx + \int_0^5 f(x) dx = 4$$

$$\int_0^5 [1 + f(x)] dx = 7 \implies \int_0^5 f(x) dx = 2$$

96. If $\lim_{x \rightarrow 0} \phi(x) = a^2$, where $a \neq 0$, then

what is $\lim_{x \rightarrow 0} \phi\left(\frac{x}{a}\right)$ equal to?

- (a) a^2
- (b) a^{-2}
- (c) $-a^2$
- (d) $-a$

$$\int_{-2}^0 f(x) dx + 2 = 4$$

97. What is $\lim_{x \rightarrow 0} e^{-\frac{1}{x^2}}$ equal to?

- (a) 0
- (b) 1
- (c) -1
- (d) Limit does not exist

98. If A is a square matrix, then what is $\text{adj}(A^{-1}) - (\text{adj}A)^{-1}$ equal to?

- (a) $2|A|$
- (b) Null matrix
- (c) Unit matrix
- (d) None of the above

99. What is the binary equivalent of the decimal number 0.3125?

- (a) 0.0111
- (b) 0.1010
- (c) 0.0101
- (d) 0.1101

100. Let R be a relation on the set N of natural numbers defined by ' $nRm \iff n$ is a factor of m '. Then which one of the following is correct?

- (a) R is reflexive, symmetric but not transitive
- (b) R is transitive, symmetric but not reflexive
- (c) R is reflexive, transitive but not symmetric
- (d) R is an equivalence relation

101. What is $\int_0^{4\pi} |\cos x| dx$ equal to?

- (a) 0
- (b) 2
- (c) 4
- (d) 8

$$2 \int_0^{2\pi} |\cos x| dx$$

102. What is the number of natural numbers less than or equal to 1000 which are neither divisible by 10 nor 15 nor 25?

- (a) 860
- (b) 854
- (c) 840
- (d) 824

$$\frac{(10+k+1) - 7}{2} = 7$$

$$\frac{10+k+1}{2} = 7$$

$$10+k+1 = 14$$

$$k = 3$$

(a, 2b) (k, 4)

(10, -b)

103. (a, 2b) is the mid-point of the line segment joining the points (10, -6) and (k, 4). If $a - 2b = 7$, then what is the value of k?

- (a) 2
- (b) 3
- (c) 4
- (d) 5

$$a + 2b = 7$$

$$\frac{10+k}{2} = a$$

$$k = 2a - 10$$

$$\frac{-6+4}{2} = 2b$$

$$-1 = 2b$$

$$b = -\frac{1}{2}$$

$$a - 2(-\frac{1}{2}) = 7$$

$$a + 1 = 7$$

$$a = 6$$

$$k = 2(6) - 10 = 12 - 10 = 2$$

104. Consider the following statements.

1. If ABC is an equilateral triangle, then $3 \tan(A+B) \tan C = 1$.
2. If ABC is a triangle in which $A = 78^\circ, B = 66^\circ$, then

$$\tan\left(\frac{A}{2} + C\right) < \tan A$$

3. If ABC is any triangle, then

$$\tan\left(\frac{A+B}{2}\right) \sin\left(\frac{C}{2}\right) < \cos\left(\frac{C}{2}\right)$$

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) 1 and 2
- (d) 2 and 3

105. If $A = (\cos 12^\circ - \cos 36^\circ)(\sin 96^\circ + \sin 24^\circ)$ and $B = (\sin 60^\circ - \sin 12^\circ)(\cos 48^\circ - \cos 72^\circ)$, then what is $\frac{A}{B}$ equal to?

- (a) -1
- (b) 0
- (c) 1
- (d) 2

For the next four (4) items that follow :

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function such that

$$f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$$

for $x \in \mathbb{R}$.

106. What is $f(1)$ equal to?

- (a) -2
- (b) -1
- (c) 0
- (d) 4

$f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$

107. What is $f''(1)$ equal to?

- (a) -6
- (b) -5
- (c) 1
- (d) 0

108. What is $f'''(10)$ equal to?

- (a) 1
- (b) 5
- (c) 6
- (d) 8

109. Consider the following :

1. $f(2) = f(1) - f(0)$
2. $f''(2) - 2f'(1) = 12$

Which of the above is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

For the next three (3) items that follow :

A plane P passes through the line of intersection of the planes $2x - y + 3z = 2$, $x + y - z = 1$ and the point $(1, 0, 1)$.

110. What are the direction ratios of the line of intersection of the given planes?

- (a) $\langle 2, -5, -3 \rangle$
- (b) $\langle 1, -5, -3 \rangle$
- (c) $\langle 2, 5, 3 \rangle$
- (d) $\langle 1, 3, 5 \rangle$

111. What is the equation of the plane P ?

(a) $2x + 5y - 2 = 0$

(b) $5x + 2y - 5 = 0$

(c) $x + z - 2 = 0$

(d) $2x - y - 2z = 0$

112. If the plane P touches the sphere $x^2 + y^2 + z^2 = r^2$, then what is r equal to?

(a) $\frac{2}{\sqrt{29}}$

(b) $\frac{4}{\sqrt{29}}$

(c) $\frac{5}{\sqrt{29}}$

(d) 1

For the next two (2) items that follow :

Consider the function

$$f(x) = |x^2 - 5x + 6|$$

$$f'(x) = \frac{2x-5}{|2x-5|}$$

113. What is $f'(4)$ equal to?

(a) -4

(b) -3

(c) 3

(d) 2

114. What is $f''(2.5)$ equal to?

(a) -3

(b) -2

(c) 0

(d) 2

For the next two (2) items that follow :

Let $f(x)$ be the greatest integer function and $g(x)$ be the modulus function.

115. What is $(g \circ f)\left(-\frac{5}{3}\right) - (f \circ g)\left(-\frac{5}{3}\right)$ equal to?

(a) -1

(b) 0

(c) 1

(d) 2

116. What is $(f \circ f)\left(-\frac{9}{5}\right) + (g \circ g)(-2)$ equal to?

(a) -1

(b) 0

(c) 1

(d) 2

For the next two (2) items that follow :

Consider a circle passing through the origin and the points (a, b) and $(-b, -a)$.

117. On which line does the centre of the circle lie?

(a) $x + y = 0$

(b) $x - y = 0$

(c) $x + y = a + b$

(d) $x - y = a^2 - b^2$

118. What is the sum of the squares of the intercepts cut off by the circle on the axes?

(a) $\left(\frac{a^2 + b^2}{a^2 - b^2}\right)^2$

(b) $2\left(\frac{a^2 + b^2}{a - b}\right)^2$

(c) $4\left(\frac{a^2 + b^2}{a - b}\right)^2$

(d) None of the above

For the next two (2) items that

Let \hat{a}, \hat{b} be two unit vectors and θ be the angle between them.

119. What is $\cos\left(\frac{\theta}{2}\right)$ equal to?

(a) $\frac{|\hat{a} - \hat{b}|}{2}$

(b) $\frac{|\hat{a} + \hat{b}|}{2}$

(c) $\frac{|\hat{a} - \hat{b}|}{4}$

(d) $\frac{|\hat{a} + \hat{b}|}{4}$

120. What is $\sin\left(\frac{\theta}{2}\right)$ equal to?

(a) $\frac{|\hat{a} - \hat{b}|}{2}$

(b) $\frac{|\hat{a} + \hat{b}|}{2}$

(c) $\frac{|\hat{a} - \hat{b}|}{4}$

(d) $\frac{|\hat{a} + \hat{b}|}{4}$